Lesson_11

- Conversions between map coordinates of different projections

Conversions between map coordinates

The engineering and the earth sciences apply large data sets of map coordinates coming from different ages and land-surveying, furthermore relating to different geodetic datums and map projections, which causes difficulties by the integration of data. Therefore the conversion of map coordinates to another one is often required. A type of conversions utilizes the concerning reference surfaces and projections – these are the *coordinate methods*. An other type of conversions takes neither the reference surfaces nor the projections into consideration, only the location of the points, and they are called *conversions by polynomials*.

Conversions by the coordinate methods

Inverse projection equations are applied for the calculation from the source map coordinates to the geographic coordinates, and the *direct projection equations* of an other projection for the calculation from the geographic coordinates to the target map coordinates.

The coordinate methods can be classified whether the geodetic datums and projections of the *source* and *target* coordinate systems are identical or diverse.

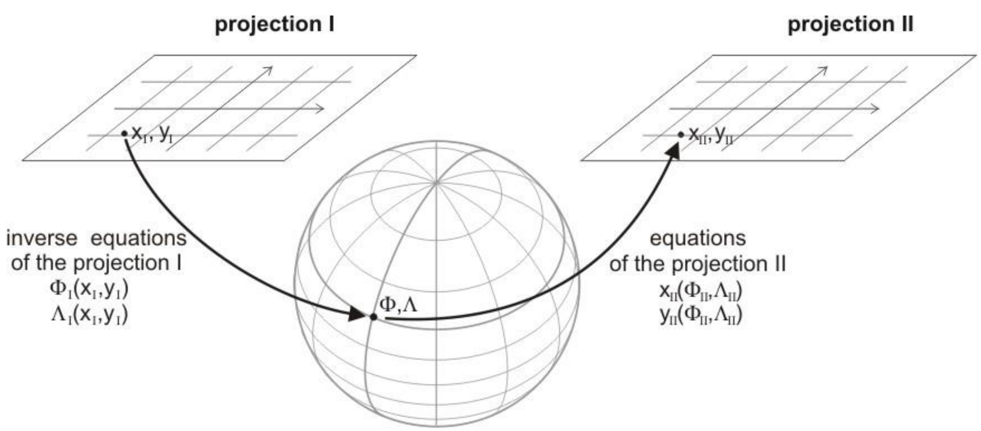
- **Identical datums, identical projections**

  The source and the target projections have the same equations, only the longitudes of the midmeridian or the true scale parallels differ in them. This method is appied by the neighbouring Gauss-Krüger or UTM zones and neighbouring WAC ICAO sheets.

Conversions by the coordinate methods (2)

- **Identical datums, different projections**

  The ellipsoidal coordinates derive from the source map coordinates by the inverse projection equations, and then the target coordinates can be obtained by the target projection equations.



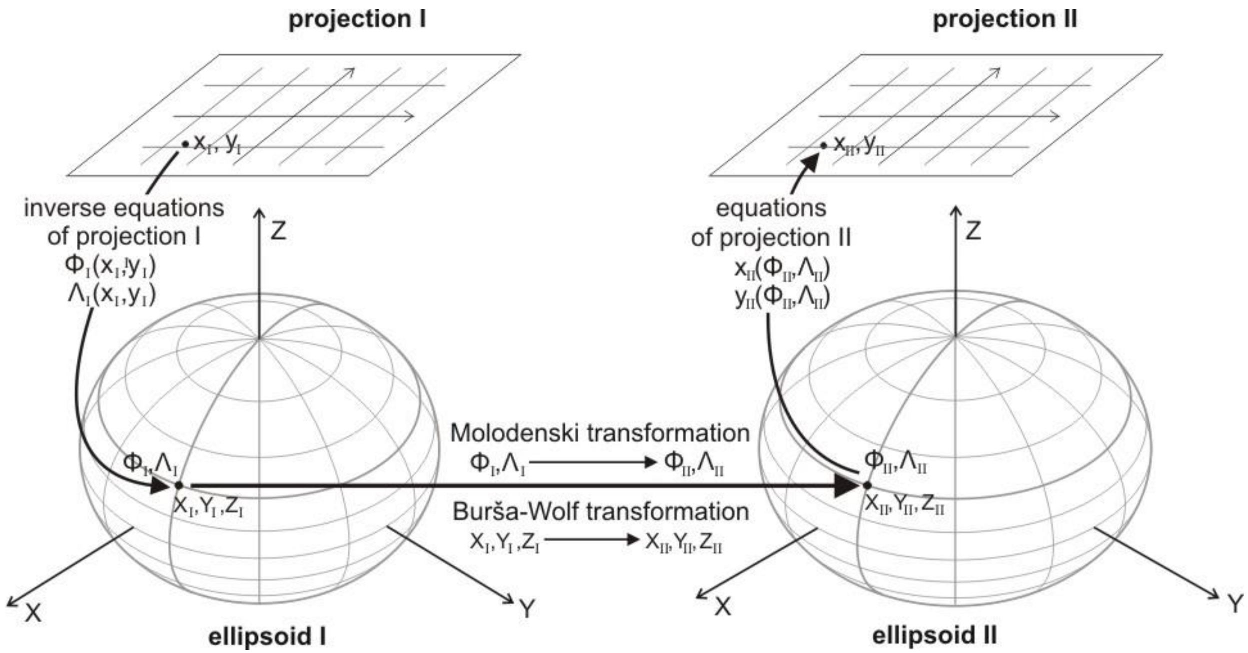
- **Different datums, identical projections**

  If two neigbouring countries use the same projection for their map system, but with different geodetic datum, then on the area near the boundary, the inverse projection equations provide the ellipsoidal coordinates, which can be converted onto the other datum (e.g. either by the Molodenski or the Burša-Wolf transformation), finally the target projection equations give the target coordinates.

Conversions by the coordinate methods (3)

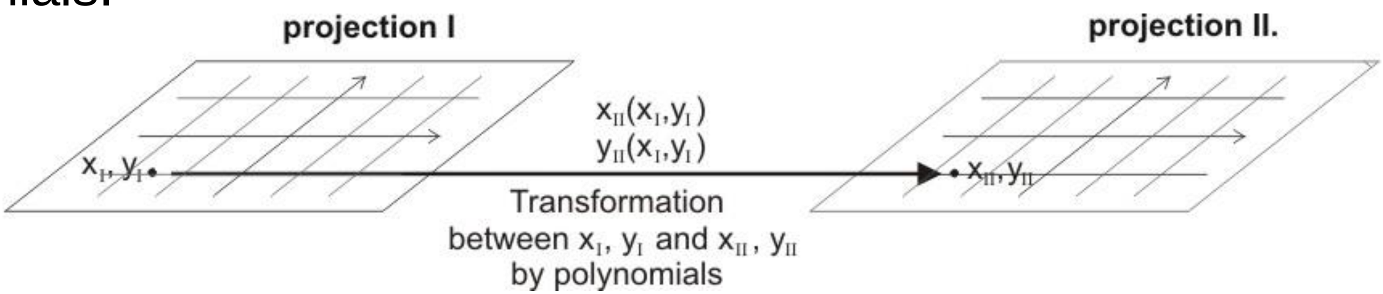
- **Different datums, different projections**

  In most of the practical cases this general version occurs. After calculating the geographic coordinates on the ellipsoid from the source map coordinates by the inverse projection equations, a conversion of these ellipsoidal coordinates onto the other geodetic datum follows with help of fitting points, finally the projection equations give the target map coordinates.



Conversion by polynomials (1)

Regardless of the mathematical relationships between the source and target map coordinates and the correspondent geographic coordinates, it is possible to carry out simplified conversion between the map coordinates by polynomials.



Denoting the source map coordinates by x_I, y_I and the target map coordinates by x_{II}, y_{II}, the connections between the correspondent coordinates are given as functions

$$x_{II}(x_I, y_I)$$
$$y_{II}(x_I, y_I)$$

which can be approximated by polynomials of the source map coordinates.

The approximation needs *fitting points* with their correspondent map coordinates, and the error depends partly on the degree of the polynomials.

Conversion by polynomials (2)

Conversion by linear polynomials:

$$\hat{x}_{II} = A_0 + A_1 \cdot x_I + A_2 \cdot y_I$$

$$\hat{y}_{II} = B_0 + B_1 \cdot x_I + B_2 \cdot y_I$$

where A_0, A_1, A_2 and B_0, B_1, B_2 are constant coefficients, and their determination needs 3 fitting points.

Substituting the correspondent map coordinates in the equations above, 3 linear equations can be obtained for both coefficients A_j and B_j (j=0,1,2):

$$x_{II\,i} = A_0 + A_1 \cdot x_{I\,i} + A_2 \cdot y_{I\,i} \qquad (i=1,2,3)$$

$$y_{II\,i} = B_0 + B_1 \cdot x_{I\,i} + B_2 \cdot y_{I\,i} \qquad (i=1,2,3)$$

The solution of these systems of equations provides the coefficients A_j and B_j, and so the polynomials of the conversion. The resulted points x_{II}, y_{II}, calculated from an arbitrary point x_I, y_I, will be accurate only at the fitting points, and the error usually increases moving away from them on the source map plane.

The accuracy of conversion can be improved by raising the degree of the polynomials.

Conversion by polynomials (3)

The quadratic approaching polynomials are the following:

$$\hat{x}_{II} = A_0 + A_1 \cdot x_I + A_2 \cdot y_I + A_3 \cdot x_I^{\,2} + A_4 \cdot x_I \cdot y_I + A_5 \cdot y_I^{\,2}$$

$$\hat{y}_{II} = B_0 + B_1 \cdot x_I + B_2 \cdot y_I + B_3 \cdot x_I^{\,2} + B_4 \cdot x_I \cdot y_I + B_5 \cdot y_I^{\,2}$$

Here 6 fitting points are required for the determination of the coefficients A_j and B_j (j=0,1,2,…,5). Substituting the correspondent coordinates into these equations, two linear systems of equations can be got for the coefficients:

$$x_{II\,i} = A_0 + A_1 \cdot x_{I\,i} + A_2 \cdot y_{I\,i} + A_3 \cdot x_{I\,i}^{\,2} + A_4 \cdot x_{I\,i} \cdot y_{I\,i} + A_5 \cdot y_{I\,i}^{\,2} \qquad \text{(i=1,2,…,6)}$$

$$y_{II\,i} = B_0 + B_1 \cdot x_{I\,i} + B_2 \cdot y_{I\,i} + B_3 \cdot x_{I\,i}^{\,2} + B_4 \cdot x_{I\,i} \cdot y_{I\,i} + B_5 \cdot y_{I\,i}^{\,2} \qquad \text{(i=1,2,…,6)}$$

The result of the conversion of an arbitrary point x_I, y_I given by this equations is accurate at the fitting points, but the error at other points can be smaller as in the linear case. The degree of the approximating polynomials can be raised further, but the required number of the fitting points grows, too:

degree of the polynomials	1	2	3	4	5
required number of the fitting points	3	6	10	15	21

The further increase of the degree of polynomials is not recommended.

In case of higher number of fitting points, the method of *least sum of squares* is in use.

Conversion by polynomials (4)

Example:

Y_I=526746.19m $\quad$ Y_{II}=?

X_I=451696.73m $\quad$ X_{II}=?

Fitting points:

Y_{I1}=318819.83m $\quad$ Y_{II1}=468839.43m

X_{I1}=525620.59m $\quad$ X_{II1}=263693.08m

Y_{I2}=528945.56m $\quad$ Y_{II2}=678949.86m

X_{I2}=356048.79m $\quad$ X_{II2}= 94167.41m

Y_{I1}=732355.33m $\quad$ Y_{II1}=882345.18m

X_{I1}=594541.41m $\quad$ X_{II1}=332603.99m

Estimating formulae: $Y_{II} \approx B_0+B_1 * X_I+B_2 * Y_{II}$

$X_{II} \approx A_0+A_1 * X_I+A_2 * Y_{II}$

(A_0, A_1, A_2, and B_0, B_1, B_2 are the solutions of a linear system of equations).

Conversion by polynomials (5)

The system of equations concerning A_j :

$A_0+A_1*525620.59+A_2*318819.83 = 263693.08$

$A_0+A_1*356048.79+A_2*528945.56 = 94167.41$

$A_0+A_1*594541.41+A_2*732355.33 = 332603.99$

Its solution e.g. by the *Cramer's rule*:

$A_0=0.2214999438*10^{17}/(-0.846059967*10^{11})=-261801.71$

$A_1=(-0.845848379*10^{11})/(-0.846059967*10^{11})=0.99974991$

$A_2=(-0.14989*10^{7})/(-0.846059967*10^{11})= 0.00001772$

The estimation of X_{II}: $X_{II} \approx -261801.71+0.99974991*X_I+0.00001772*Y_I=$
$= 189791.4\text{m}$

The real value of X_{II}: $X_{II} \approx 189807.14\text{m}$

Home work:

to establish and solve the system of equations concerrning the coefficients B_i.