Introduction

The Earth (or any other celestial body) is an irregular shape. Most often it is represented by a sphere or an [ellipsoid of revolution] (a.k.a. spheroid). (There are several minor celestial bodies that are represented by [triaxial ellipsoid] due to their very irregular shape.)

Maps depict the surface of the Earth on a plane. Map projections define the rules of mapping the curved surface into the plane by assigning a place on the plane for every single point of the curved surface.

Coordinate systems on the sphere and the ellipsoid of revolution.

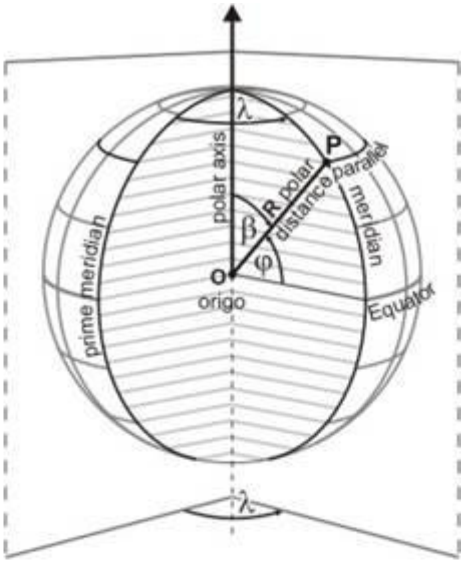

Figure 1. Geographic coordinate system on the sphere.

Figure 1 represents the geographic coordinate system on the sphere. It is a 3D polar coordinate system. Its centre is the centre of the sphere. The position of a **P** point on the sphere surface can be described by a pair of angles: φ and λ, or β and λ. The R distance between the centre of the sphere (**O**) and the point **P** is constant and equal to the sphere radius for any point on the sphere surface. The geographic coordinates are:

λ – *geographic longitude*: the angle between the plane of the prime meridian and the plane containing the poles and point P. Lines of constant longitude are called *meridians*. The longitudes can be values between ±180°. Negative values are often referred as *western*, while positive values as *eastern* longitudes. Be aware that on extraterrestrial bodies the range of longitudes can be different (0–360°). Longitude values depend on the position of the prime meridian. Nowadays the standard prime meridian is the one crossing the Greenwich observatory in London, but there were several other prime meridians throughout in the history.

φ – *geographic latitude*: the angle between the OP line and the plane of the Equator (which is the plane perpendicular to the polar axis and containing the centre of the sphere). The value of latitudes is between ±90°. Positive values are referred as *northern*, negatives as *southern* latitudes. Circles of constant latitude are called *parallels*. The parallel of latitude 0° is called Equator.

β – polar angle: the angle between the OP line and the polar axis. The value of the polar angle is between 0 and 180°. The connection between φ and β is: $\beta = 90° – \varphi$. Polar angle is mostly used instead of latitude when discussing azimuthal projections as the formulas are more straightforward that way.

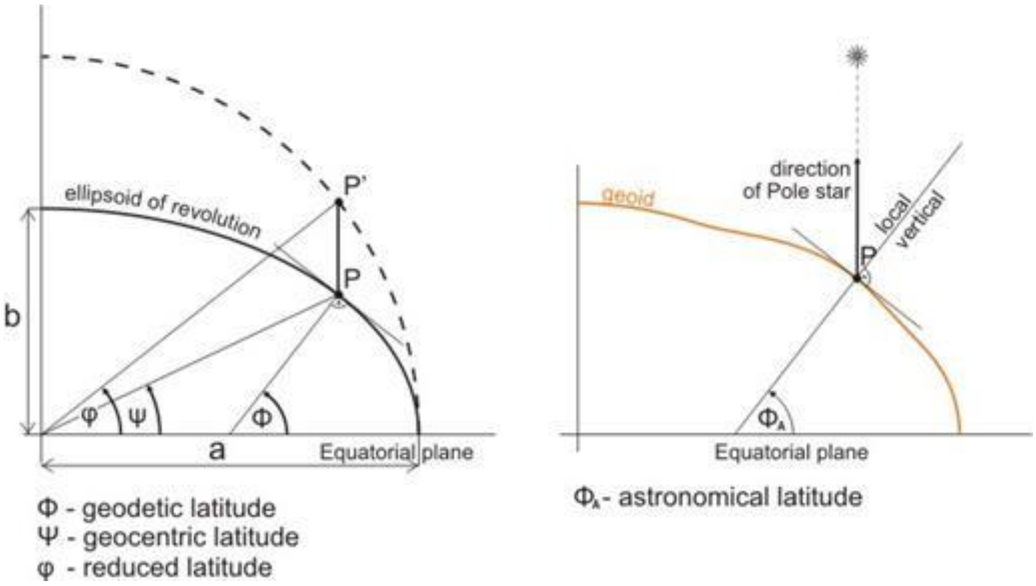

Figure 2. Various latitudes on the ellipsoid of revolution and the geoid.

An ellipsoid of revolution can be described with its major and minor half axes (a and b, see Figure 2). Due to the effect of the centrifugal force, the axis of rotation is always the minor axis. **Figure 2** shows the various latitudes on an ellipsoid. The most common one is the Φ geodetic latitude; the angle between the plane of the Equator and the local vertical line (perpendicular to the surface). Similarly to the sphere, B polar angle can be calculated as $B = 90° - \Phi$.

Longitudes are marked by the Greek letter Λ, and are the same on the ellipsoid as λ on the sphere (Figure 3).

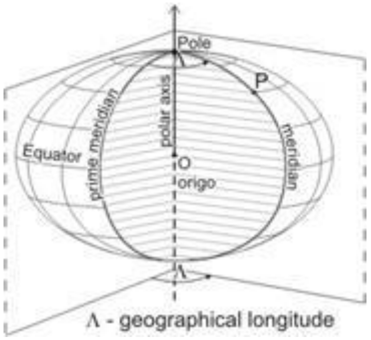

Figure 3. Geographic longitude on the ellipsoid of revolution.

Coordinate systems on the plane

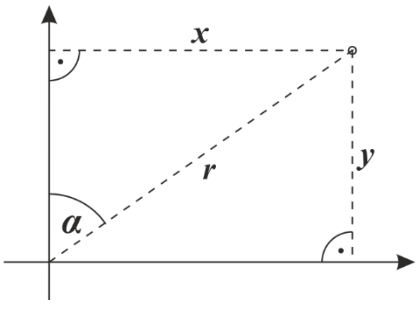

Figure 4. Cartesian and polar coordinate systems on the plane.

Points on a plane are described either by Cartesian (x, y) or polar (r, α) coordinates (Figure 4). Conversion formulas between the two systems are:

$$x = r \sin \alpha$$

$$y = r \cos \alpha$$

$$r = \sqrt{x^2 + y^2}$$

$$\cos \alpha = \frac{y}{r}$$

Projection equations (formulas)

Projection equations or formulas map (φ, λ) [or alternatively (β, λ) or (Φ, Λ) or (B, Λ)] to (x, y), and are usually expressed with two mathematical functions:

$x = x(\varphi, \lambda)$ and

$y = y(\varphi, \lambda)$.

Expected properties of the projections:

- describable by formulas,
- injective (one-to-one),
- twice continuously differentiable.