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Determining the Projection of Small Scale Maps Based on Graticule Line Shapes

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Summary: When determining the projection of an unknown (especially large scale) map, the usual approach is a numerical estimation: defining a multi-variable projection function and finding a parameter set which results in minimum error at the specified control points.
The approach described in this paper is rather different. Small scale maps depicting the whole world, or continents show sections of latitude or longitude lines long enough to determine their curve type (straight lines, circle, ellipse or other curve sections). The curve type of geographic grid lines and their spacing together usually determines the projection.
This paper introduces a "decision tree" – a set of yes or no questions about the grid lines of the map that can help determining the projection of a small scale map especially for ones who are not experts of this field.

Introduction

The digital processing of a scanned map usually involves georeferencing. Although there are georeferencing methods not requiring the exact definition of the projection of the map, these techniques produce results with considerable errors, or require a very dense network of control points in order to avoid these errors. Knowing the projection of the map may significantly increase the accuracy of georeferencing and decrease the number of control points needed.
Unfortunately, there are several cases when the map projection is unknown – it is not indicated on the map, neither in other sources. There are various solutions to solve this case, usually based on numerical estimation of the projection formulas: defining multi-parameter functions (e.g. quadratic, cubic or higher degree polynomials), and finding a parameter set which results in minimum error at the specified control points (e.g. in Tobler 1966). Another approach is to calculate the position of a set of control points in several projections and to choose the best fitting one, e.g. in (Bayer 2008).
The following approach is different. The shape of the geographic grid depends on the map projection; therefore, it is possible to deduce the projection (or at least to define a small group of possible candidates) by simply observing specific characteristics of longitude and latitude lines.
Naturally, this method is only applicable to small scale maps, displaying the whole Earth or at least entire continents. Medium or large scale maps usually display only small sections of the geographic grid that is insufficient for determining the curve type.
This way of recognising projections has a long tradition in Hungarian cartography. Érdi-Krausz (1958) wrote very detailed paper on projection analysis, describing measuring methods, and distortion calculation methods, as well as a detailed classification of projections. He defined 12 main projection groups based on the shape of grid lines. Györffy (2012), following similar

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principles, created 11 groups – some groups matching exactly the ones from Érdi-Krausz's system, while others do not.
The main groups described in this paper are matching the ones of Györffy. However, as some of these groups contain rather rarely used projections, the order is rearranged: easily recognisable and widely used projection groups are in the first part of the list, and Györffy's "others" group was divided into three further groups.

Naming conventions of projections and projection groups

Map projections may have several different names. Not counting the translations of the same phrase to various languages, the same projection may have different names in various ages and areas (e.g. Mercator-Sanson, Sanson-Flamsteed or Sinusoidal is the same projection).
Even worse the case of the projection groups. Various scientists created their own classification and the same term may refer to different groups. As an example, pseudoconic projections are a small group in Snyder's system, but contain the group of polyconic projections according to Györffy.
The projection names in this paper are mostly those that they have in various GIS software or in the book (Bugayevskiy, Snyder 1995). Group names are also taken from this book.

Recognising projections

In general, it is much easier to recognise projections in *normal aspect*. The curvature of the geographic grid in *transverse* or *oblique* aspect of a particular projection is usually totally different from the normal aspect, therefore these cases are much harder to detect – although the unique shape of the oblique or transverse form of a few specific projections makes them easily recognisable.

Observations and measurements

It is very important to differentiate latitude (parallel) and longitude (meridian) lines. If there is any doubt, one should look for grid numbering at the edge of the map; usually there is a sign or abbreviation referring to Eastern/Western (longitudes) or Northern/Southern (latitudes).
The *central meridian* can be arbitrarily chosen in most projections. The central meridian is usually easily recognisable on the map, being the vertical axis of symmetry of the geographic grid. Note that the central meridian is not necessarily identical to the *prime meridian*.
Several projections have one or more *equidistant* (either latitude or longitude) grid lines (i.e. linear distortion is uniform along the lines). This characteristic can be easily observed: grid intersections along equidistant grid lines are equally spaced. *True scale* lines do not distort distances; the proportion of distances measured along these lines on the map and on the Earth are equal to the map scale.
The *spacing* of grid lines is always important. Parallels or meridians may have *uniform* spacing, or it can be increasing or decreasing towards a specific direction.
We should always observe the angle of meridians and parallels, especially whether it is right angle or not.

Sometimes the radii of specific circles is needed for classification. If the centre of a circle is not indicated on the map, it can be deduced geometrically by drawing two chords of the circle and their perpendicular bisector. The intersection of the perpendiculars is the circle centre.

Projection groups

The first step of projection recognition is deciding which main group the projection fits into. Further analysis (mostly of the spacing of grid lines) allows the differentiation of several additional subgroups. These groups are the following:

1. <u>Longitudes are straight lines intersecting in one specific point; latitudes are closed circles.</u>

These are **azimuthal projection**s in **polar aspect**. If only a fragment of the curvature is visible, it is advisable to check whether the angle of two meridians is equal to the difference of their longitudes. If their angle is less than this difference, then the projection falls to group 2.
There are three subgroups in this group, based on the spacing of parallel circles.

- 1/a. If spacing of parallels is uniform, then it is the *Azimuthal Equidistant* projection.
- 1/b. If spacing is decreasing with the distance from the projection centre, the projection is probably *Lambert Azimuthal Equal-Area*, or eventually *ortographic*. In the case of ortographic projection the spacing decreases faster, and only one hemisphere is visible, while – theoretically – Lambert Azimuthal is capable of displaying the whole Earth in one map.
- 1/c. If the space between parallels grows with the distance from the centre, it is probably the Stereographic Projection, or eventually Gnomonic. In this latter case spacing grows very fast – Gnomonic projection is applicable to only areas much smaller than a half hemisphere, therefore it is rather scarcely used in cartographic practice.

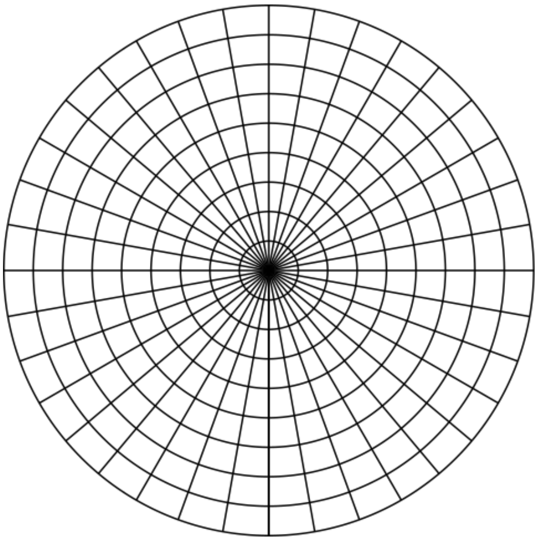
Although there are other azimuthal projections as well, they are rarely used.

2. <u>Longitudes are straight lines intersecting in one specific point; latitudes are partial circle arcs.</u>

This is the group of **conic projections**. Further grouping is based on the spacing of latitude lines.

- 2/a. If the spacing of parallels is uniform, it is Equidistant Conic.
- 2/b. If the spacing is descending with the distance from the pole, it is Lambert (if the pole is a point) or Albers (if the pole is a circle arc) Equal-area Conic projection
- 2/c. If the spacing is growing with the distance from the centre, it is probably Lambert Conformal Conic projection. (Perspective Conic projection would also fall into this group but it is not used in cartographic practice.)

Additional parameters are the latitudes of standard parallels, which can be calculated from the angle of meridians and the radius of the pole line in each case, (Györffy 2012).



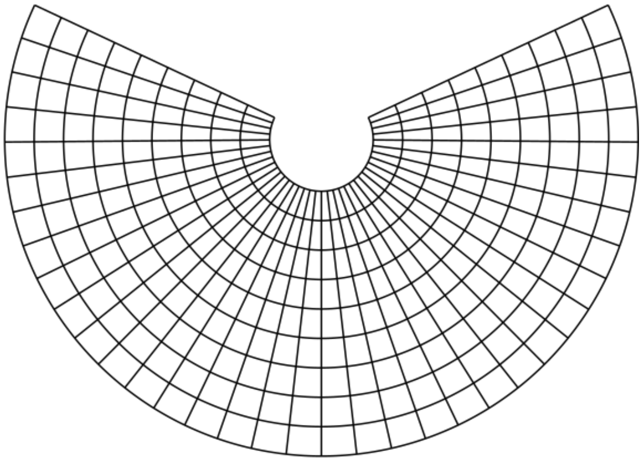


Figure 1: Samples of group 1 (left, Azimuthal Equidistant) and 2 (right, Equidistant Conic).

3. <u>Longitudes and latitudes are straight lines, perpendicular to each other.</u>

These are **cylindrical projections** in **normal aspect**. Actual projections depend on spacing of parallels:

- 3/a. If the spacing is uniform, it is the Equirectangular projection. The true scale latitude (φ_N) can be calculated by measuring the aspect ratio of a uniform-sided (e.g. 10°×10°) geographical quadrangle: the ratio is equal to cos φ_N.
- 3/b. If the spacing decreases towards the poles, the projection is probably Lambert Cylindrical Equal-Area (including Hobo-Dyer and Gall-Peters). The true-scale latitude (φ_N) can be calculated by measuring the dimensions of a given (φ_1, φ_2),(λ_1, λ_2) quadrangle: $\cos\varphi_N = \frac{\sin\varphi_2 - \sin\varphi_1}{\lambda_2 - \lambda_1}\frac{180°}{\pi}\frac{dx}{dy}$, where λ_1, λ_2 are in degrees and $\frac{dx}{dy}$ is the aspect ratio of the quadrangle.
- 3/c. If the spacing grows towards the poles, it is probably the Mercator projection (it also could be Perspective Cylindrical but that one is very rarely used).

4. <u>Latitudes are straight parallel lines, while longitudes are various curves.</u>

This is the group of **pseudocylindrical projections**. There are several projections in this group therefore it is advisable to observe the following characterictics: the shape of longitude lines (circle arcs, ellipse arcs, sine arcs or straight lines), the poles (whether they are points or lines), the spacing of parallels.

- 4/a. If longitudes are straight lines, it can be
  - the “Donis” projection of Nicolaus Germanus (15th century), if the poles are points and the spacing of latitudes is uniform, or
  - Collignon projection if the poles are points but the spacing of latitudes is not uniform.
  - Eckert’s 1st or 2nd projection, if the poles are lines. Eckert’s 1st has uniform spacing, while the 2nd not.
- 4/b. If longitude pairs are full ellipses (and thus poles are points), depicting the whole Earth in a 2:1 aspect ellipse, then depending on the spacing of parallels, it can be Apian’s 2nd projection (uniform spacing), or Mollweide (non-uniform spacing). If only a hemisphere is depicted in a circle, it is the orthographic projection in equatorial aspect.

- 4/c. Longitudes are ellipse parts (not full ellipses), poles are lines. If the longitudes join to the pole lines smoothly, it is Eckert 3rd (with uniform parallel spacing) or 4th (non-uniform spacing) projection, otherwise Kavrayskiy VII.
- 4/d. Longitudes are circle arcs. If only a half hemisphere is in the circle, then it is Apian's 1st projection. If the whole Earth fits to a circle, it is probably Van der Grinten's 3rd projection.
- 4/e. Longitudes are sine arcs. If the poles are points and the spacing of parallels is uniform, it is the Mercator-Sanson. If the poles are lines, then it is probably Eckert's 5th (uniform parallel spacing) or 6th (non-uniform parallel spacing) projection. It also can be Kavrayskij's 6th projection, which is rather similar to Eckert's 6th.
- 4/f. If the longitudes are none of the previous, it is probably a composite pseudocylindrical projection. (Unfortunately composite projections are not widely supported by GIS systems.)

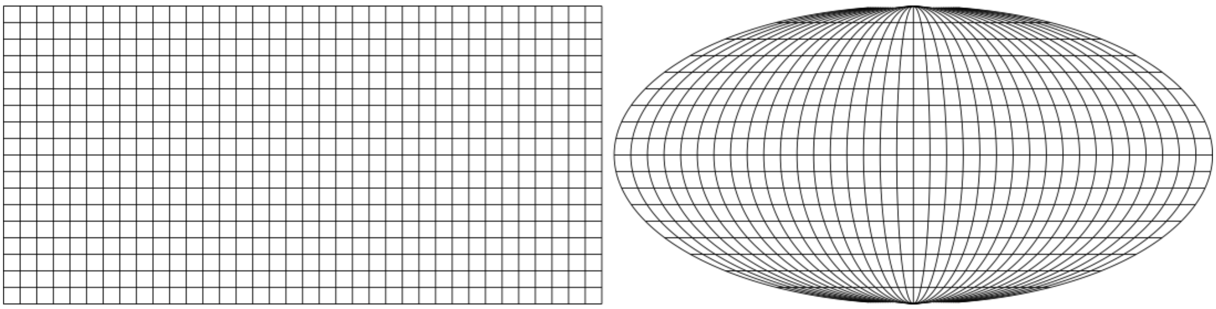


Figure 2. Samples of group 3 (left, Equirectangular) and 4 (right, Apian II).

5. <u>Latitudes are partial circle arcs while longitudes are other curves.</u>

This is the group of **pseudoconic and polyconic projections**.

- 5/a. If the latitudes are partial concentric circles, it is a *pseudoconic* projection. The most often used one is the *Bonne projection*, in which the central meridian and latitude lines are equidistant. This projection may get various shapes depending on the setting of normal latitude. (Normal latitude can be observed on the map; this is the only latitude arc, which is perpendicular to all meridians). The *Stabius-Werner projection* is a special case of Bonne with normal latitude set to 90°.
- 5/b. The latitudes are *non-concentric* partial circles, and their radius is proportional to the cotangent of latitude. These are the "true" polyconic projections. If all the parallels are true scale lines, it is the "Ordinary" (or American) Polyconic projection. If the central meridian is equidistant, then if the latitudes are always perpendicular to longitudes, it is the Orthogonal Polyconic, otherwise probably the Equal-area polyconic projection. If all the meridians are straight lines, it is the "Modified Polyconic" projection for the 1 : 1 000 000 scale International Map of the World. If none of the above, and the spacing along the central meridian increases towards the poles, it is probably the conformal polyconic projection.
- 5/c. The latitudes are *non-concentric* partial circles; the Equator is equidistant. If the whole world fits to a circle, it is *van der Grinten's 1st* or *2nd* projection – longitudes and latitudes are perpendicular in the 2nd version. If the world fits to a shape of two intersecting circles, it is van der *Grinten's 4th*.

- 5/d. The latitudes are *non-concentric* partial circles; the Equator is *not equidistant*; longitude and latitude lines are perpendicular. If the whole world is depicted, it is Lagrange's projection (the whole Earth can be either a circle or a shape of two intersecting circles, depending on projection parameters). If only smaller areas are shown (maximum a hemisphere), it also can be the *stereographic projection in oblique or equatorial aspect*.

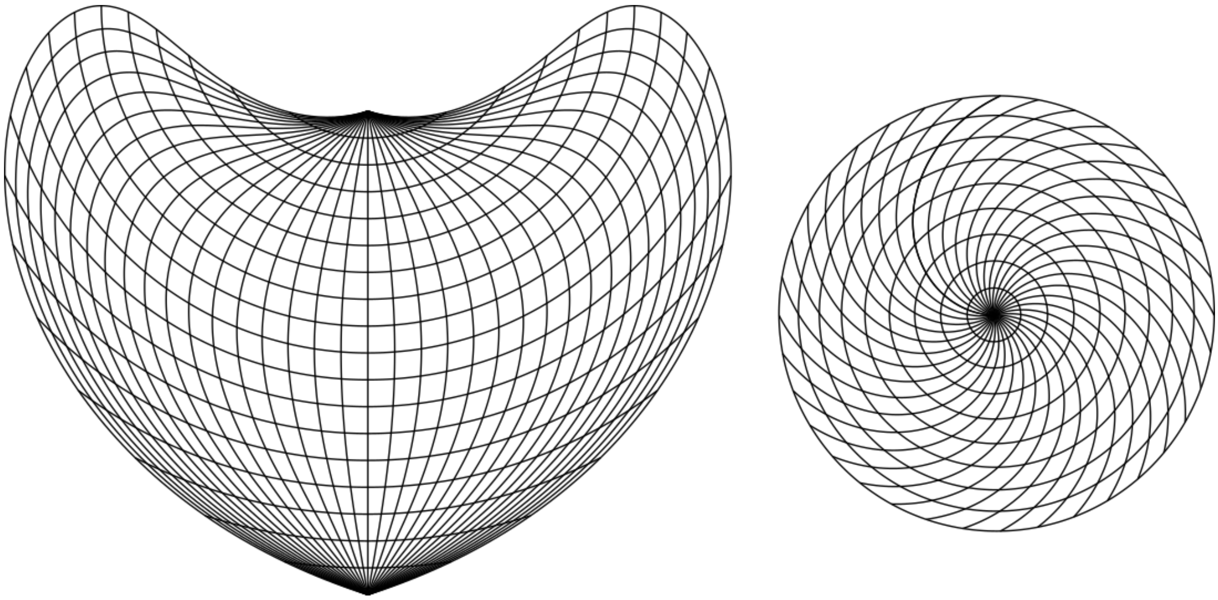


Figure 3. Samples of group 5 (left, Bonne projection) and 6 (right, Wiechel projection)

6. <u>Latitudes are full circles while longitudes are other curves.</u>

This is the group of **pseudoazimuthal projections** in polar aspect. These projections are very rarely used; one example is *Wiechel's projection*, an equal-area projection in which meridians are uniform circle arcs. Another example is Ginzburg's *TsNIIGAiK projection.* In this projection, the 0° – 180° and the -90° – 90° meridians are perpendicular straight lines, while all the other longitudes are curves having their concave side towards the 0° – 180° meridian pair.

7. <u>Longitudes are parallel straight lines, while latitudes are hyperbolas.</u>

This is the gnomonic projection in equatorial aspect. (rarely used)

8. <u>Longitudes are straight lines, meeting in one common point, latitudes are various conic sections (ellipses, parabolas or hyperbolas).</u>

This is the gnomonic projection in oblique aspect. (rarely used)

9. <u>All grid lines are ellipses or ellipse arcs.</u>

If a hemisphere is depicted in a circle, then it is the orthographic projection in oblique aspect. Erwin Raisz's "Armadillo" projection also belongs to this group.

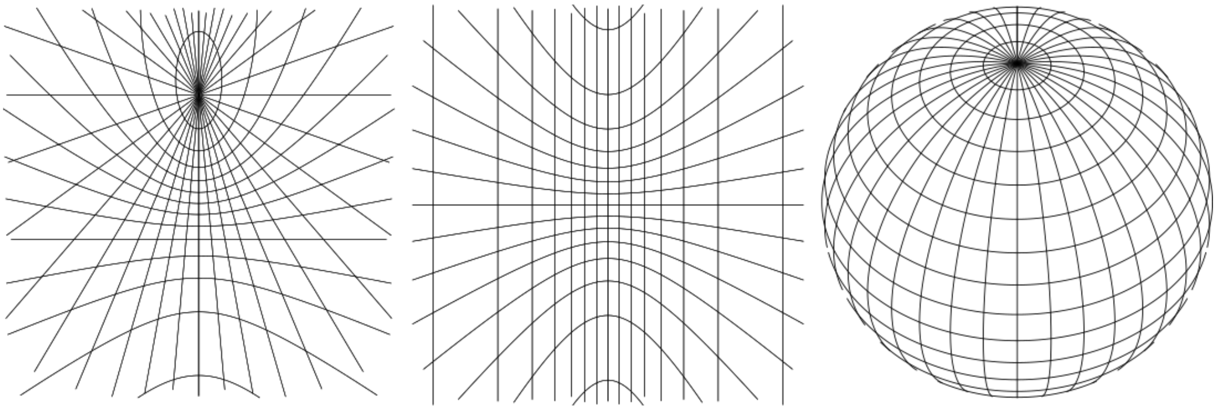


Figure 4. Samples of group 7 (left, oblique gnomonic), 8 (middle, transverse gnmonic) and 9 (right, orthographic).

10. Latitudes are ellipses, longitudes are hyperbolas.

These are experimental projections with little practical use. The most common is the Littrow Projection.

11. Others (i.e. none of the previous groups) with two axes of symmetry (the central meridian and the equator).

- 11/a. The whole Earth is shown in a 2:1 aspect ellipse: It is either the Hammer or Aitoff projection.
- 11/b. A half hemisphere is shown in a circle: **azimuthal projections** in **equatorial aspect**. If the Equator and the central meridian are equidistant, it is *Azimuthal Equidistant* projection; if their spacing is descending towards the outline of the map then *Lambert Azimuthal Equal-Area*, otherwise *stereographic* projection, all in equatorial aspect.
- 11/c. Poles are straight lines: it is probably the Winkel Tripel projection
- 11/d. Poles are arcs: Hammer-Wagner (Wagner VII) or Wagner XI projection

12. Other projections with the central meridian as the only axis of symmetry.

Most of the cylindrical, conical and azimuthal projections in oblique aspect fall into this group. Further classification requires complex measurements.

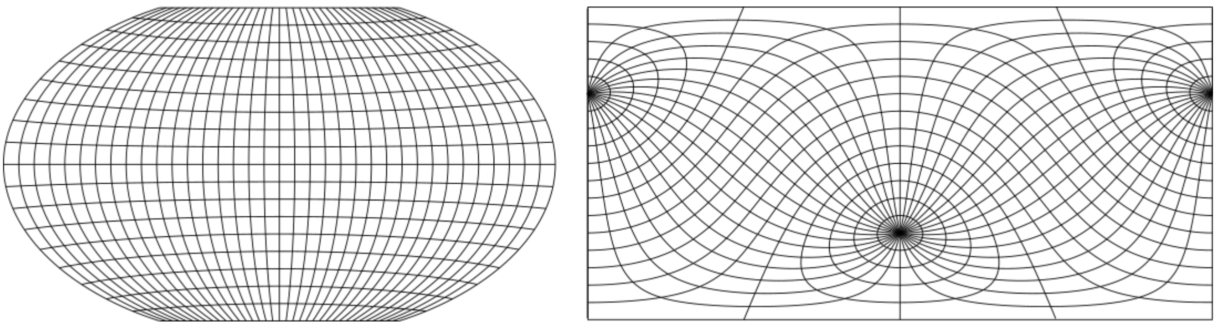


Figure 5. Samples of group 11 (left, Winkel Tripel) and 12 (right, Equirectangular projection in oblique aspect).

13. All the others.

Circles, ellipses, other curves

The classification above requires the differentiation of circle, ellipse arcs, sometimes even parabola or hyperbola curves. While it is quite simple to recognise the curve type in the case of full circles or ellipses, it is rather difficult when only partial arcs are visible.
In order to help recognizing curve types, the authors are developing a software tool which determines curve types of digitized grid lines.
This tool uses a semi-automatic approach for determining projections. Line-drawing tools are provided on a web-based frontend for manually tracing graticule lines on pre-uploaded raster maps. Users also have to supply the traced line's geographical coordinates.
Given the approximate traces of graticule lines, we employ a number of iterative least-squares fitting algorithms in order to determine the most probable class the curve fits into. Currently we are able to differentiate between circular, elliptic, parabolic, hyperbolic arcs and straight lines. (The first four can be fit in a single step as they are all types of conic sections.)
In least-squares fitting, the goodness of the fit is proportional to the offsets of trace points from the curve – irrespective of curve type and the exact algorithm used. This means our method can be easily extended (e.g. by recognizing sinusoidal curves to take Mercator-Sanson and Eckert's 5th and 6th projections into account).
Identifying the shape of graticule lines, however provides only a partial solution to our problem.
As mentioned earlier, one should further observe the properties and relationships of graticule lines (e.g. equidistance, concentricity, their angle of intersection etc.) to be able to precisely place the projection in the hierarchy we outlined previously.
Having computed the above parameters, one can utilize a decision-tree mechanism to obtain the exact projection type. As a final step, further, projection-specific methods can be developed to compute projection parameters and emit georeferencing metadata associated with the raster map in question.

Conclusion, further development possibilities

The previously described classification can help to determine the (unknown) projection – or at least the projection group – of small scale maps. Naturally, knowing the name of a projection not always enough for georeferencing as there may be one or more parameters that affect the shape of the map such as true scale latitudes. The estimation of these parameters require different approaches in each case. As this classification is primarily designed for "non-cartographers", these calculations are included only in a few, simple cases.
The most promising development possibility is to create a software that can detect map projection with minimal user interaction, following the scheme described previously. The first step could be the automatic vectorisation of grid lines. It can be followed by the determination of curve types (whether they are lines or circle arcs etc.). This information can be used to determine the projection group; and finally projection parameters could be calculated from the mathematical parameters of the digitised curves.

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